Journal of Engineering Mathematics (2005) 51: 317–324
DOI 10.1007/s10665-004-1770-9

Ferrofluid squeeze film between curved annular plates including rotation of magnetic particles

RAJESH C. SHAH and M.V. BHAT[1]
Department of Mathematics, Nirma Institute of Technology, Sarkhej-Gandhinagar Highway, Ahmedabad 382 481, Gujarat, India (E-mail: dr_rcshah@yahoo.com); [1]E-202, Riddhi Complex, Near Jodhpur Village, Ahmedabad 380 015, Gujarat, India

Received 7 October 2002; accepted in revised form 28 July 2004

Abstract. The effects of ferrofluid on the curved squeeze film between two annular plates, when the upper plate approaches the lower one normally, are studied including the rotation of the magnetic particles and their magnetic moments. The aim is to study the effects of rotation of the magnetic particles on the characteristics of the squeeze film. The main equation is derived in the Appendix A. Expressions for the pressure, load capacity and response time are obtained. Load capacity and response time are found to increase when the volume concentration of the solid phase, Langevin's parameter or the curvature of the upper plate are increased.

Key words: curved annular plates, ferrofluid, lubrication, rotation of magnetic particles

1. Introduction

The study of a squeeze film between flat annular plates is a classical one [1, pp. 258–297]. Vibrations in jet engines can be absorbed using annular squeeze films between engine bearings and their support. A pair of clutch plates may be modelled by two annular plates [2, pp. 85–101]. Owing to elastic, thermal and uneven wear effects the plates encountered in practice are not actually flat. In this work we consider the case of an exponentially curved upper plate as studied, for example, by Murti [3]. Recently, many theoretical investigations were made using a ferrofluid as lubricant owing to its various advantages such as long life, silent operation and reduced wear. Verma [4] initiated the study of a ferrofluid-based squeeze film. Bhat and Deheri [5] studied a squeeze film between porous annular disks. Shah *et al.* [6] extended their study to include the curvature of the upper disk and rotation of both disks. Gupta and Vora [7] analysed a squeeze film between curved annular plates. Bhat and Deheri [8] and Shah and Bhat [9] analysed a squeeze film between curved porous circular disks, in both non-rotating as well as rotating cases. Agrawal [10] studied a porous inclined slider bearing, and Bhat and Deheri [11] a porous composite slider bearing using a magnetic fluid lubricant. All found that magnetization of the fluid increased the load capacity of the bearing studied. All the authors [4–6, 8–11] used the Neuringer-Rosensweig model for a ferrofluid under a variable oblique magnetic field. However, unlike the Shiliomis model [12] which was used by Sinha *et al.* [13] to study the ferrofluid lubrication of cylindrical rollers with cavitation and by Shukla *et al.* [14] to derive the pressure equation, the Neuringer-Rosensweig model fails to give any contribution to the flow field when a constant magnetic field is used. The pressure equation derived in the Appendix A of the present work differs from that used by Shukla and Kumar [14] which is derived under the assumptions that the ferrofluid is saturated, so that the saturation magnetization does not depend upon the applied magnetic field and that the magnetic moment relaxation time is negligible. Recently Shah and Bhat [15] studied a ferrofluid squeeze film in a long journal bearing using the Shliomis, Jenkins and the Neuringer-Rosensweig model of flow.

The aim of the present work is to study the behaviour of a curved squeeze film between two annular plates with a ferrofluid lubricant under a constant transverse magnetic field using the Shliomis model.

2. Analysis

The bearing consists of two annular plates each of inside radius b and outside radius a ($a > b$). The geometry of the problem is shown in Figure 1. The film thickness h is taken as [3]

$$h = h_0 e^{-\beta r^2}, \quad b \le r \le a, \tag{1}$$

where r is the radial coordinate, h_0 is the central film thickness and β is the curvature of the upper plate. The upper plate approaches the lower with a constant normal velocity

$$\dot{h_0} = \frac{dh_0}{dt}.$$

As the calculations in the Appendix A show, if an axially symmetric flow of the ferrofluid between the plates under a constant transverse magnetic field is assumed, the film pressure p is given by (A18), namely

$$\frac{1}{r}\frac{d}{dr}\left(h^3 r \frac{dp}{dr}\right) = 12\eta_0 \left(1 + \frac{5}{2}\phi\right)(1+\tau)\dot{h_0}, \tag{2}$$

where η_0, ϕ, and τ are the viscosity of the liquid, volume concentration of the particles and the rotational viscosity parameter, respectively.

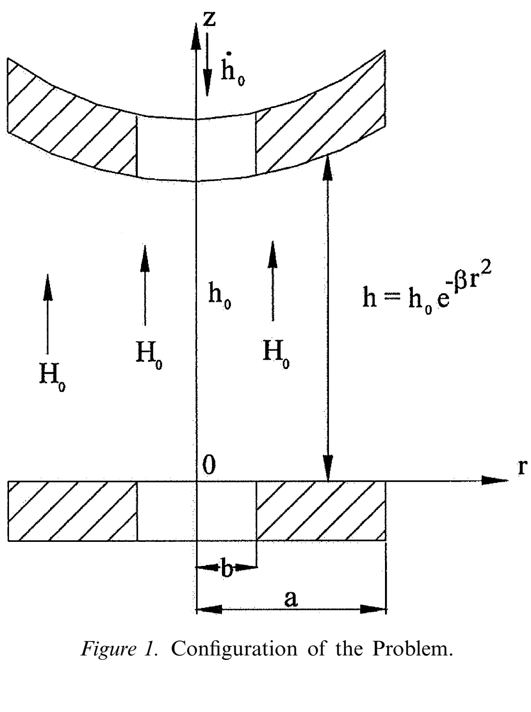

Figure 1. Configuration of the Problem.

3. Solution

Upon introduction of the dimensionless quantities

$$R=\frac{r}{b},\quad \bar{h}=\frac{h}{h_0},\quad \bar{\beta}=\beta b^2,\quad P=\frac{-h_0^3 p}{\eta_0 b^2 \dot{h}_0},\tag{3}$$

and use of (1), Equation (2) transforms to

$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}R}\left(\mathrm{e}^{-3\bar{\beta}R^2}R\frac{\mathrm{d}P}{\mathrm{d}R}\right)=-12\left(1+\frac{5}{2}\phi\right)(1+\tau).\tag{4}$$

Solving Equation (4) subject to the boundary conditions

$$P(1)=P(k)=0,\tag{5}$$

where $k=a/b$, we obtain the dimensionless pressure P as

$$P=A\int_1^R \frac{\mathrm{e}^{3\bar{\beta}R^2}}{R}\mathrm{d}R-\frac{\left(1+\frac{5}{2}\phi\right)(1+\tau)}{\bar{\beta}}\left(\mathrm{e}^{3\bar{\beta}R^2}-\mathrm{e}^{3\bar{\beta}}\right),\tag{6}$$

with

$$A=\frac{\left(1+\frac{5}{2}\phi\right)(1+\tau)\left(\mathrm{e}^{3\bar{\beta}k^2}-\mathrm{e}^{3\bar{\beta}}\right)}{\bar{\beta}\int_1^k \frac{\mathrm{e}^{3\bar{\beta}R^2}}{R}\mathrm{d}R}.\tag{7}$$

The load capacity W of the bearing can be expressed in dimensionless form as

$$\begin{aligned}\bar{W}&=-\frac{h_0^3 W}{2\pi\eta_0 b^4\dot{h}_0}=\int_1^k RP\,\mathrm{d}R=-\frac{1}{2}\int_1^k R^2\frac{\mathrm{d}P}{\mathrm{d}R}\mathrm{d}R\\&=-\frac{1}{12\bar{\beta}^2}\left[A\bar{\beta}\left(\mathrm{e}^{3\bar{\beta}k^2}-\mathrm{e}^{3\bar{\beta}}\right)-2\left(1+\frac{5}{2}\phi\right)(1+\tau)\left\{(3\bar{\beta}k^2-1)\mathrm{e}^{3\bar{\beta}k^2}-(3\bar{\beta}-1)\mathrm{e}^{3\bar{\beta}}\right\}\right].\end{aligned}\tag{8}$$

The time taken by the upper plate to reach a central film thickness h_0, starting from an initial film thickness h_1, is obtained in dimensionless form as

$$\bar{t}=\pi\bar{W}\left(\frac{1}{\bar{h}_0^2}-1\right),\tag{9}$$

where

$$\bar{h}_0=\frac{h_0}{h_1},\quad \bar{t}=\frac{h_1^2 W t}{\eta_0 b^4}.\tag{10}$$

4. Results and discussion

The expressions for the load capacity, $\bar{W}$, and time to reach a thickness h_0, $\bar{t}$, for a squeeze film between flat plates are obtained by letting $\bar{\beta}\to 0$ in (8) and (9) and are given by

$$\bar{W}=-\frac{3}{4}\left(1+\frac{5}{2}\phi\right)(1+\tau)(k^2-1)\left[\frac{k^2-1}{\log\,k}-(k^2+1)\right],\tag{11}$$

$$\bar{t}=\frac{3\pi}{4}\left(1+\frac{5}{2}\phi\right)(1+\tau)(k^2-1)\left[\frac{k^2-1}{\log\,k}-(k^2+1)\right]\left(1-\frac{1}{\bar{h}_0^2}\right),\tag{12}$$

the computed values of which correspond to the curves for $\bar{\beta}=0$ in Figures 3–6.

One can obtain the expressions for the $\bar{W}$ and $\bar{t}$ by letting $\phi = 0$ in (8–9) and (11–12) for the curved squeeze film case [7] and the classical case [1], respectively.

Expressions for dimensionless film pressure P, load capacity $\bar{W}$ and response time $\bar{t}$ of the squeeze film are given by (6), (8) and(9), respectively. They are functions of the outer-inner radii ratio k of the plate, the curvature $\bar{\beta}$ of the upper plate, the volume concentration ϕ of the magnetic particles, and Langevin's parameter ξ. The computed values of the rotational viscosity parameter τ for various values of ϕ and ξ are shown in Figure 2 which shows that τ increases with ϕ as well as ξ.

The computed values of $\bar{W}$ for various values of ϕ, ξ and $\bar{\beta}$ are shown in Figures 3 and 4. $\bar{W}$ increases with increasing values of ϕ, ξ and $\bar{\beta}$. The values of $\bar{W}$ when the upper plate is flat ($\bar{\beta} = 0$) lie between those when the upper plate is convex ($\bar{\beta} < 0$) and those when the upper plate is concave ($\bar{\beta} > 0$). The values of $\bar{t}$ for various values of ϕ, ξ and $\bar{\beta}$ are shown in Figures 5 and 6. Figures 5 and 6 are similar to Figures 3 and 4 and $\bar{t}$ behaves like $\bar{W}$. One concludes from (9) that $\bar{t}$ is less than, equal to or greater than $\bar{W}$ according to $\bar{h}_0$ being less than, equal to or greater than $\sqrt{\pi/(\pi+1)}$.

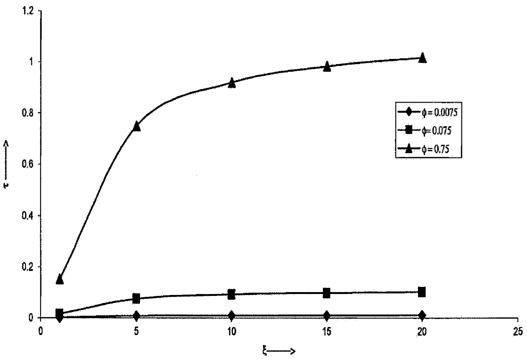

Figure 2. Values of τ for various values of ϕ and ξ.

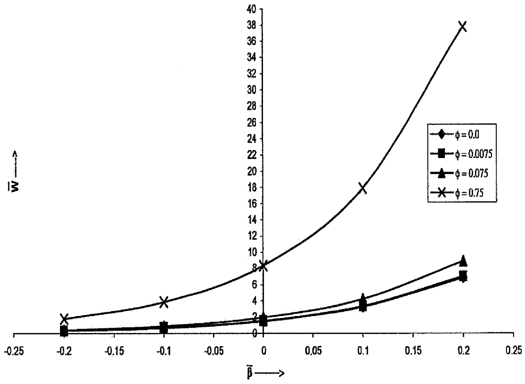

Figure 3. Dimensionless load capacity $\bar{W}$ for various value of ϕ and $\bar{\beta}$ with $\xi = 10, k = 2$.

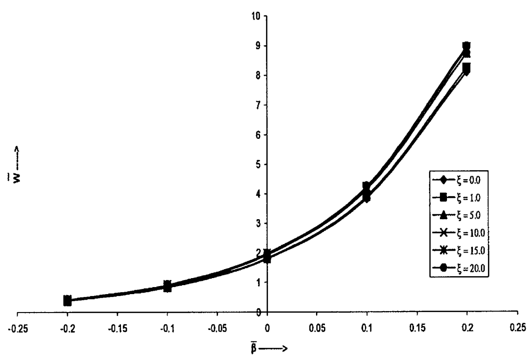

Figure 4. Dimension load capacity $\bar{W}$ for various values of ξ and $\bar{\beta}$ with $\phi = 0{\cdot}075, k = 2$.

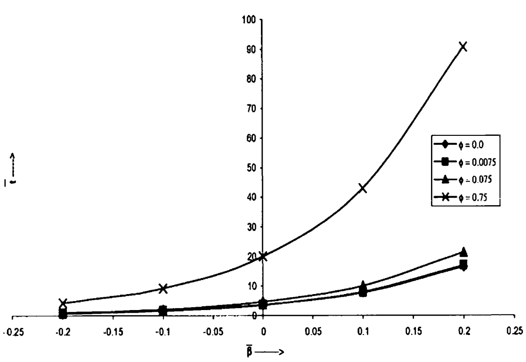

Figure 5. Dimesionless response time $\bar{t}$ for various values of ϕ and $\bar{\beta}$ with $\xi = 10, k = 2, \bar{h}_0 = 0{\cdot}75$.

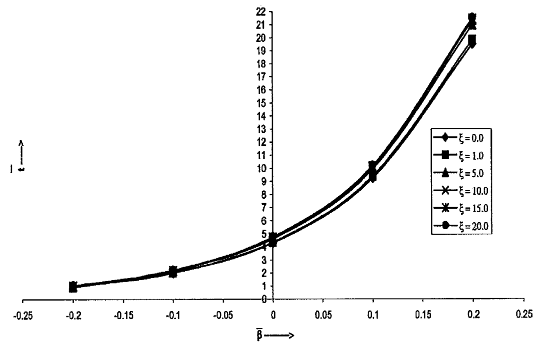

Figure 6. Dimesionless response time $\bar{t}$ for various values of ξ and $\bar{\beta}$ with $\phi = 0{\cdot}075, k = 2, \bar{h}_0 = 0{\cdot}75$.

5. Conclusions

A constant magnetic field does not enhance the bearing characteristics in the Neuringer-Rosensweig model. However, it does in the Shliomis model in which rotation of the magnetic particles and their moments are included. Increases in the rotational parameter and the curvature of the upper plate cause increases in the load capacity and response time of the annular squeeze film. The load capacity and response time of the annular squeeze film can be made optimal by a judicious choice of a concave upper plate.

Acknowledgement

The authors are grateful to the referees for their valuable suggestions.

Appendix A, Derivation of the governing equation

Following Shliomis [12], Kumar [16, p. 9] pointed out that magnetic particles of a ferrofluid can relax in two ways as the applied magnetic field changes. The first is by the rotation of magnetic particles in the fluid given by the Brownian relaxation time parameter τ_B, and the second is by rotation of the magnetic moment within the particles given by the relaxation time parameter τ_s.

However, Shukla and Kumar [14] derived the pressure equation under the assumptions that the ferrofluid is saturated so that the saturation magnetization is independent of the applied magnetic field and the magnetic moment relaxation time is negligible. We derive below the pressure equation without the above assumptions.

Assuming steady flow, neglecting inertia and the second derivative of the internal angular momentum $\bar{S}$, we obtain the following equations governing the flow [12]:

$$-\nabla p + \eta \nabla^2 \bar{q} + \mu_0 (\bar{M} \cdot \nabla) \bar{H} + \frac{1}{2\tau_s} \nabla \times (\bar{S} - I\bar{\Omega}) = 0, \tag{A1}$$

$$\bar{S} = I\bar{\Omega} + \mu_0 \tau_s (\bar{M} \times \bar{H}), \tag{A2}$$

$$\bar{M} = M_0 \frac{\bar{H}}{H} + \frac{\tau_{\mathrm{B}}}{I} (\bar{S} \times \bar{M}), \tag{A3}$$

where p is the pressure, η is the viscosity of the suspension, μ_0 is the permeability of free space, $\bar{H}$ is the applied magnetic field, $\bar{M}$ is the magnetization vector, $\bar{q}$ is the fluid velocity, I is the sum of moments of inertia of the particles per unit volume, $\bar{\Omega} = (1/2) \nabla \times \bar{q}, \tau_B$ is the

Brownian relaxation time, τ_s is the magnetic moment relaxation time, and M_0 is the equilibrium magnetization, together with the equation of continuity,

$$\nabla \cdot \bar{q} = 0, \tag{A4}$$

and Maxwell's equations

$$\nabla \times \bar{H} = 0, \tag{A5}$$

$$\nabla \cdot (\bar{H} + \bar{M}) = 0. \tag{A6}$$

Using (A2), we observe that (A1) and (A3) yield

$$-\nabla p + \eta \nabla^2 \bar{q} + \mu_0 (\bar{M} \cdot \nabla) \bar{H} + \tfrac{1}{2} \mu_0 \nabla \times (\bar{M} \times \bar{H}) = 0 \tag{A7}$$

and

$$\bar{M} = M_0 \frac{\bar{H}}{H} + \tau_B \bar{\Omega} \times \bar{M} - \frac{\mu_0 \tau_B \tau_s}{I} \bar{M} \times (\bar{M} \times \bar{H}). \tag{A8}$$

Langevin's parameter ξ is a measure of the dimensionless field strength. For a strong magnetic field $\xi > 1$. In this case τ_s can not be neglected. However, Shukla and Kumar [14] neglected τ_s in their analysis. Then (A8) can be approximated as (see [12])

$$\bar{M} = \frac{M_0}{H} [\bar{H} + \bar{\tau} (\bar{\Omega} \times \bar{H})], \tag{A9}$$

where

$$\bar{\tau} = \frac{\tau_B}{1 + \frac{\mu_0 \tau_B \tau_s}{I} M_0 H}.$$

For a suspension of spherical particles $I/\tau_s = 6\eta\phi$ and $\tau_B = 3\eta V/(k_B T)$, where $\phi = nV$ is the volume concentration of the particles, k_B is the Boltzmann constant, n is the number of particles per unit volume and T is the temperature, one can express $\bar{\tau}$ as

$$\bar{\tau} = \frac{6\eta\phi}{n k_B T (1 + \xi \coth \xi)}, \tag{A10}$$

taking

$$M_0 = n\mu (\coth \xi - 1/\xi) \quad \text{and} \quad H = k_B T \xi / (\mu_0 \mu), \tag{A11}$$

μ being magnetic moment of a particle, as in Shliomis [12]. Shukla and Kumar [14] assumed that M_0 did not depend upon ξ in contrast to (A11).

In an axially symmetric flow under a uniform magnetic field $\bar{H} = (0, 0, H_0)$ with radial velocity component u, (A7) and (A9) yield

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta \left(1 + \frac{\mu_0 M_0 H_0 \bar{\tau}}{4\eta}\right)} \frac{\mathrm{d}p}{\mathrm{d}r}. \tag{A12}$$

From (A10–A12) one obtains

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta(1+\tau)} \frac{\mathrm{d}p}{\mathrm{d}r}, \tag{A13}$$

where

$$\tau=\frac{3}{2}\phi\frac{\xi-\tanh\xi}{\xi+\tanh\xi}. \tag{A14}$$

Solving (A13) under the no-slip boundary conditions

$$u=0 \quad \text{when } z=0,h,$$

one obtains

$$u=\frac{z^2-hz}{2\eta(1+\tau)}\frac{\mathrm{d}p}{\mathrm{d}r}. \tag{A15}$$

Substituting (A15) in the integal form of the continuity equation

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\int_0^h ru\,\mathrm{d}z+\dot{h}_0=0$$

yields

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(h^3 r\frac{\mathrm{d}p}{\mathrm{d}r}\right)=12\eta(1+\tau)\dot{h}_0. \tag{A16}$$

If η_0 is the viscosity of the main liquid, the viscosity of the suspension is given by the Einstein formula [12]

$$\eta=\eta_0\left(1+\tfrac{5}{2}\phi\right). \tag{A17}$$

Equations (A16–A17) yield

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(\mathrm{h}^3 r\frac{\mathrm{d}p}{\mathrm{d}r}\right)=12\eta_0\left(1+\frac{5}{2}\phi\right)(1+\tau)\dot{\mathrm{h}}_0. \tag{A18}$$

References

1. M.M. Khonsari and E.R. Booser, *Applied Tribology: Bearing design and Lubrication*. New York: John Wiley & Sons, Inc. (2001) 496 pp.
2. B.C. Majumdar, *Introduction to Tribology of Bearings*. New Delhi: Wheeler Publishing (1999) 327 pp.
3. P.R.K. Murti, Squeeze film in curved circular plate. *J. Lubr. Technol*. 97 (1975) 650–652.
4. P.D.S. Verma, Magnetic fluid-based squeeze film. *Int. J. Engng. Sci*. 24 (1986) 395–401.
5. M.V. Bhat and G.M. Deheri, Squeeze film behaviour in porous annular discs lubricated with magnetic fluid. *Wear* 151 (1991) 123–128.
6. R.C. Shah, S.R. Tripathi and M.V. Bhat, Magnetic fluid based squeeze film between porous annular curved plates with the effect of rotational inertia. *Pramana-J. Phys*. 58 (2002) 545–550.
7. J.L. Gupta and K.H. Vora, Analysis of the squeeze film between curved annular plates, *J. Lubr. Technol.* 102 (1980) 48–50.
8. M.V Bhat and G.M. Deheri, Magnetic-fluid-based squeeze film in curved porous circular discs. *J. Magn. Magn. Mater*. 127 (1993) 159–162.
9. R.C. Shah and M.V Bhat, Squeeze film based on magnetic fluid in curved porous rotating circular plates. *J. Magn. Magn. Mater*. 208 (2000) 115–119.
10. V.K. Agrawal, Magnetic fluid-based porous inclined slider bearing. *Wear* 107 (1986) 133–139.
11. M.V. Bhat and G.M. Deheri, Porous composite slider bearing lubricated with magnetic fluid. *Jap. J. Appl. Phys*. 30 (1991) 2513–2514.
12. M.I. Shliomis, Effective viscosity of magnetic suspensions. *Sov. Phys. JETP* 34 (1972) 1291–1294.
13. P. Sinha, P. Chandara and D. Kumar, Ferrofluid lubrication of cylindrical rollers with cavitation. *Acta Mech*. 98 (1993) 27–38.

14. J.B. Shukla and D. Kumar, A theory of ferrofluid lubrication *J. Magn. Magn. Mater*. 65 (1987) 375–378.
15. R.C. Shah and M.V Bhat, Ferrofluid squeeze film in a long journal bearing. *Tribol. Int.* 37 (2004) 441–446.
16. D. Kumar, *Contributions to the Theory of Ferrofluid Lubrication*. Ph.D. Thesis, IIT Kanpur (Dec. 1989) 189 pp.